Session-2018-19

CARNOT CYCLE

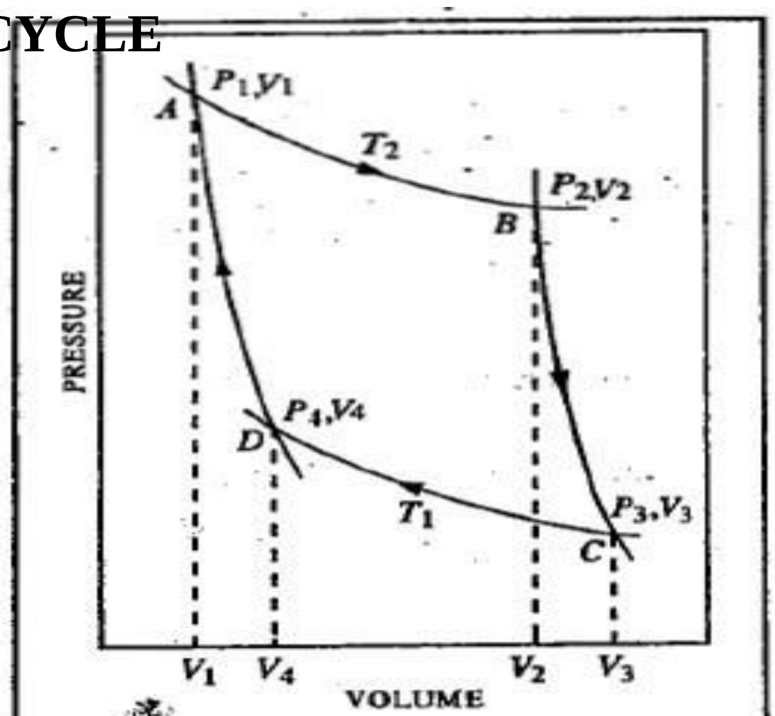

Fig. 1. The four operations in Carnot cycle.

I. Stroke 1. Isothermal Expansion.

$$q_2 = -w_1 = RT_2 \ln (V_2/V_1) \qquad \ldots(1)$$

II. Stroke 2. Adiabatic Expansion.

Now, by definition, $C_v = (\partial U/\partial T)_v$ ...(2)

$$\therefore \quad \Delta U = C_v \, \Delta T = C_v(T_1 - T_2) = -w \quad ...(3)$$

(change in temperature, ΔT = final temperature - initial temperature)

$$\text{or} \quad -w = C_v(T_1 - T_2) = -C_v(T_2 - T_1) \quad ...(4)$$

If the work done in this stage is denoted by w_2, then

$$\therefore \quad -w_2 = -C_v(T_2 - T_1) \quad ...(5)$$

III. Stroke 3. Isothermal Compression.

$$-q_1 = w_3 = RT_1 \ln (V_4/V_3) \quad \ldots(6)$$

IV. Stroke 4. Adiabatic Compression.

$$\Delta U = w = C_v\Delta T = C_v(T_2 - T_1)$$

Let w_4 be the work done in this stage. Then,

$$w_4 = C_v(T_2 - T_1) \quad \text{...(7)}$$

The net heat absorbed (q) by the ideal gas in the whole cycle is given by

$$q = q_2 + (-q_1) = RT_2 \ln (V_2/V_1) + RT_1 \ln (V_4/V_3)$$
$$= RT_2 \ln (V_2/V_1) - RT_1 \ln (V_3/V_4)$$

$$C_v \ln (T_2/T_1) = R \ln (V_3/V_2) \quad \text{(For stage II)}$$

$$C_v \ln (T_2/T_1) = R \ln (V_4/V_1) \quad \text{(For stage IV)}$$

$$V_3/V_2 = V_4/V_1 \quad \text{or} \quad V_2/V_1 = V_3/V_4$$

$$q = q_2 - q_1 = R\,(T_2 - T_1) \ln (V_2/V_1)$$

Similarly, the net work done by the gas is given by

$$w = -w_1 + (-w_2) + w_3 + w_4$$

$$= RT_2 \ln V_2/V_1 - C_v(T_2 - T_1) + RT_1 \ln (V_4/V_3) + C_v(T_2 - T_1)$$

$$= RT_2 \ln (V_2/V_1) - RT_1 \ln (V_3/V_4)$$

Since $\quad V_2/V_1 = V_3/V_4$

Hence, $\quad w = R(T_2 - T_1) \ln (V_2/V_1)$

$$q_2 = RT_2 \ln (V_2/V_1)$$

12 by Eq. 1,

$$w = q_2\,(T_2 - T_1)/T_2$$

Efficiency of a Heat Engine. *The fraction of the heat absorbed by an engine which it can convert into work gives the* **efficiency** (η) *of the engine.*

From Eq. 13, it is seen that

$$\text{Efficiency, } \eta = w/q_2 = (T_2 - T_1)/T_2 \qquad \ldots(14)$$