

Subjects of Ph.D. Course Work

S. No.	Nature	Subject Code	Subject Name	Teaching Scheme (L-T-P-J)	Credits
1	Compulsory	PREC 0001	RESEARCH METHODOLOGY	3-0-1-0	4
2	Compulsory	PREC 0010	RESEARCH AND PUBLICATION ETHICS	2-0-0-0	2
Bouq	uet of Electiv	ves (as per tl	ne research area):		
1	Electives	PMAE 0010	COMPUTER SKILLS, TENSOR AND RIEMANNIAN GEOMETRY	4-0-0-0	4
1		PMAE 0011	RELATIVITY AND COSMOLOGY	4-0-0-0	4
2	Electives	PMAE 0020	INVENTORY MODELING	4-0-0-0	4
		PMAE 0021	RANDOM VARIABLES AND SOFTWARE TOOLS	4-0-0-0	4
3	Electives	PMAE 0030	PARTIAL DIFFERENTIAL EQUATIONS: METHODS AND APPLICATIONS	4-0-0-0	4
5		PMAE 0031	FRACTIONAL DIFFERENTIAL EQUATIONS	4-0-0-0	4
4	Electives	PMAE 0040	COMPUTER BASED NUMERICAL TECHNIQUES AND SOFT COMPUTING	4-0-0-0	4
		PMAE 0041	APPLIED STATISTICS AND OPTIMIZATION TECHNIQUES	4-0-0-0	4
5	Electives	PMAE 0050	DIFFERENTIAL GEOMETRY OF MANIFOLDS	4-0-0-0	4
		PMAE 0051	RIEMANN-FINSLER GEOMETRY	4-0-0-0	4
		PMAE 0060	FLUID DYNAMICS AND SOFTWARE TOOLS	4-0-0-0	4
6	Electives	PMAE 0061	COMPUTATIONAL METHODS FOR FLUID DYNAMICS	4-0-0-0	4
7	Electives	PMAE 0070	FOUNDATIONS OF COMPLEX ANALYSIS-I	4-0-0-0	4
		PMAE 0071	FOUNDATIONS OF COMPLEX ANALYSIS-II	4-0-0-0	4

SYLLABUS FOR Ph.D. COURSEWORK (Common for all branches of Engineering, Physics and Mathematics Departments) RESEARCH METHODOLOGY (PREC 0001)

Credits: 04

L-T-P: 3-0-1

Module No.	Contents	Teaching Hours	
	Introduction to Research: Definitions, objective and characteristics of research; Types of research; Criteria of good research, challenges during any research work. Research awareness.	4	
т	Scientific/ Technical Writing Skills: The principles of conducting high quality research and composing pieces of technical writings. Writing strategies for a variety type of manuscripts, including presentation slides, statement of purpose, and popular science articles.	4	
I	Literature review: Uses of literature review; Source of information; Organization of Information. Assignment : Submit (in Excel) the literature review of your field at least 20 refereed journal (SCI/ Scopus) publications. Measuring Research Output: Citations, h-index, i10 index, impact factor		
	Research Ethics: IPR, Plagiarism and Similarity Index. UGC Regulation 2016 for M. Phil and Ph. D.	4	
П	Topic Selection: Problem identification; Criteria for prioritizing problems for research. Work Planning : Major components and outline of the different phases in a research process; Summary of the major components of a research proposal.	4	
	Basics of Statistics: Average, Measures of Skewness and Kurtosis Probability Distributions, Concepts of Correlation and Regression, hypothesis testing.		
	Scientific Tools:		
	Hands on training on Technical writing tool Latex, Microsoft Visio/ SPSS, R	2+3+3	
	Soft Computing methods implementation on MATLAB	4	
	Assignment : Submit (in Latex) a research paper of your proposed research topic at least 2 pages.	2	

Outcome: Understand the basic philosophy of research Able to do systematic Literature review Able to write a research proposal/ Synopsis Able to write a research paper, technical report, Thesis, etc.

References: Kothari C R and Garg Gaurav "Research Methodology: Methods and Techniques", New Age International

Course Syllabus Research and Publication Ethics: PREC 0010

Credits: 02

Semesters I

L-T-P: 2-0-0

Objectives:

- The course aims to develop a philosophical understanding.
- It develops the logical understanding and ethical decision making
- It expands the rationality and critical thinking to improvise the problem-solving.
- It is to cultivate the scientific temperament, philosophical approach and ethical attitude which are much needed for quality researches.
- It is to give an exposure to the scholars of various fields to understand the philosophy of research, publication ethics, understanding scientific conduct and publication misconduct.
- It is to enable scholars to understand various concepts relating open access publishing, ethical tools, using research database and metrics.

Module	Content	Teaching
No.		Hours
RPE 01	Philosophy and Ethics Introduction to Philosophy: definition, nature and scope, concept, branches	4 hrs
	Ethics: Definition, moral philosophy, nature of moral judgments and reactions.	
RPE02	Scientific Conduct Ethics for science and research, Intellectual honesty and research integrity, Scientific misconducts: Falsification, Fabrication and Plagiarism (FFP), Redundant publications: duplicate and overlapping publications, salami slicing, Selective reporting and misrepresentation of data	4 hrs
RPE03	Publication Ethics	7 hrs
	Publication ethics: definition, introduction and importance	
	Best practices/standards-setting initiatives and guidelines: Committee on Publication Ethics (COPE):, World Association of Medical Editors (WAME) etc.	
	Conflicts of interest, Publication misconduct: Definition, concept, problems that lead to unethical behavior and vice versa, types, Violation of publication ethics, authorship and contributorship, Identification of publication misconduct, complaints and appeals, Predatory publishers and journals	
RPE04	Open Access Publishing (4hrs)	4 hrs
	Open access publications and initiatives, SHERPA/RoMEO online resource to check publisher copyright & self-archiving policies, a	

	Software tool to identify predatory publications developed by SPPU: UGC-CARE list of journals, Journal finder/journal suggestion tools viz. JANE, Elsevier Journal Finder, Springer Journal Suggester, etc.	
REP05	 Publication Misconduct, Group discussions (2 hrs) Subject-specific ethical issues, FFP, authorship, Conflicts of interest, Complaints and appeals: examples and fraud from India and abroad, Software tools (2 hrs) Use of reference management software like Mendeley, Zotero etc. and anti-plagiarism software like Turnitin, Urkund 	4 hrs
REP06	Databases and research metrics (7 hrs) Databases (4hrs): Indexing databases, Citation databases: Web of Science, Scopus etc. Research Metrics (3hrs): Impact factor of a journal as per Journal Citation Report, SNIP, SJR, IPP, Cite-Score, Metrics: h- index, g-index,i-10 index, altmetrics	7hrs

Outcomes:

- The scholars will be much competent in understanding the ideas of research.
- They will develop a philosophical and ethical attitude to understand the importance of the research.
- They will have the scientific temperament to deal with the research issues.
- They will avoid misconduct in research and publication.
- It will enable them to use various tools, software, databases and research metrics.
- They will be informed, scientific and ethical in their research attitude and will be equipped with the specialized knowledge to use software and tools.

References:

- 1. Think: A Compelling Introduction to Philosophy, by Simon Blackburn.
- 2. <u>https://www.youtube.com/watch?v=IvwkMxgahA4</u>
- 3. Logical Positivism: <u>https://www.youtube.com/watch?v=VAqicPNeKYo</u>
- 4. <u>https://www.youtube.com/watch?v=1A_CAkYt3GY</u>
- 5. <u>https://www.slideshare.net/LorraineSiscar/nature-of-philosophy</u>
- 6. https://plato.stanford.edu/entries/vienna-circle/
- 7. <u>https://plato.stanford.edu/entries/logical-empiricism/</u>
- 8. https://lucidphilosophy.com/1019-2/
- 9. <u>https://www.slideshare.net/clhendricksbc/judith-jarvis-thomson-the-trolley-problem</u>
- 10. Publication Misconduct, <u>https://www.youtube.com/watch?v=fGgwNClHyCo</u>
- 11. Publication Ethics & Scientific Writing https://www.youtube.com/watch?v=QsMylCHYm4E
- 12. Open Access Publishing: https://www.youtube.com/watch?v=n1ZySivYQ3w



PMAE 0010: COMPUTER SKILLS, TENSOR AND RIEMANNIAN GEOMETRY

Course Objectives: To make the students understand the concept of tensors and Riemannian geometry and know the software packages by giving more emphasis to their applications.

Credits: 04

Semester I

L-T-P: 4-0-0

Module	Contents	Teaching
No.		Hours
		(Approx.)
	<u>Computer Skills I</u> : Introduction to Latex, MATHEMATICA	
	and MAPLE.	
	Tensor and Riemannian Geometry I: Tensor calculus,	
	n-dimensional space V _n , Superscript and subscript,	
	Transformation of coordinates, Transformation law of tensor,	
	Product of two tensors, Contraction, Trace of a tensor, Quotient	
	law, Metric tensor and Riemann space.	
	Computer Skills II: Introduction to Excel, Chart, Functions.	
	Power point presentation, Introduction to C language.	
	Tensor and Riemannian Geometry II: Associated and	
	Reciprocal or conjugate tensor, Symmetric and anti symmetric	
	tensor, Tensor density, Levi-Civita tensor, Christoffel symbols,	
Ι	Law of transformation of Christoffel symbols, Covariant	45
1	differentiation, Riemannian Affine connection, Covariant	
	derivative of a vector density, Riemannian metric, Geodesics,	
	Null geodesics, Tensor form of gradient, divergence, Laplacian	
	and curl.	
	Tensor and Riemannian Geometry III: Intrinsic derivative,	
	Riemannian and normal coordinates, Gaussian coordinates,	
	Parallel transport, Geodesics as auto parallel curves, Parallel	
	propagation, Riemann curvature tensor R ⁱ ljk, Covariant curvature	
	tensor, Symmetric properties of R ⁱ _{ljk} , Covariant curvature tensor	
	R _{hljk} , Number of independent components of R _{hljk} , Ricci tensor,	
	Bianchi identities, Conformal curvature tensor, Algebraic	
	classification of the conformal curvature tensor, Conformal	
	invariance, Geodesic deviation, Lie derivative.	

Learning Outcomes:

After studying these topics, the students will be able to

- Use software packages in research
- Learn different types of tensors
- > Find Christoffel symbols and use them in the computation of different curvature tensors
- > Understand covariant differentiation, Bianchi identities and their applications

Text Books:

- S. R. Roy and Raj Bali, Theory of Relativity, Jaipur Publishing House, 2008.
- S. K. Srivastava, General Relativity and Cosmology, PHI Pvt. Ltd., 2008.
- > Leslie Lamport, A Document preparation System: Latex, Addison-Wesley Professional, 1994.
- Y. K. Singh and B. B. Chaudhary, MATLAB Programming, PHI, 2007.

Reference Book:

J. V. Narlikar, Cosmology, Cambridge University Press, 2002.



Head of the Department Mathematics Institute of Applied Sciences & Humanities GLA University, Mathura

PMAE 0011: RELATIVITY AND COSMOLOGY

Course Objectives: To make the students understand the concept of general relativity and cosmology by giving more emphasis to their applications.

Credits	: 04 Semester II	L-T-P: 4-0-0
Module No.	Contents	Teaching Hours (Approx.)
Ι	Special Theory of Relativity: Inertial and non-inertial frames, Special and General Galilean transformations, Lorentz transformation and its geometrical interpretation, Transformation formula for mass, density, momentum, energy and force, Minkowski-space, Relativistic equation of motion, Four vectors and tensors in Minkowski space, Lagrangian and Hamiltonian formulation of Relativistic Mechanics. General Relativity: Principles of equivalence and general covariance, Mach's Principle, Einstein's field equations, Energy momentum tensors, Gravitational equations, Vectors and tensors, Experimental tests of general relativity, Alternatives theories of gravitations in Brans-Dicke Theory, Kaluza's five dimensional theory, Cosmological models, Singularity in cosmological models. Cosmology: Static cosmological models, Newtonian cosmology, Einstein universe, Expanding universe, Friedmann models, Cosmological models with non-zero cosmological term, The early universe, The inflationary universe, Primordial black holes, Dark energy and dark matter, Observational constraints on cosmological parameters, Standard cosmology.	45

Learning Outcomes:

After studying these topics, the student will be able to

- > Understand the basic principles of cosmology
- > Know the significance the Einstein's theories of special and general relativity
- > Deal with the cosmological models
- > Learn various theories of gravitation

Text Books:

- S. R. Roy & Raj Bali, Theory of Relativity, Jaipur Publishing House, 2008.
- > J. V. Narlikar, An Introduction to Cosmology, Cambridge University Press, 2002.

- S. Weinberg, Cosmology, Oxford University Press, 2008.
- S. K. Srivastava, General Relativity and Cosmology, PHI Pvt. Ltd., 2008.



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PMAE 0020: INVENTORY MODELING

Course Objectives: To make the students understand the various inventory models with deterministic & probabilistic demands, price breaks and different demand functions.

Credits: ()4
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Semester I

L-T-P: 4-0-0

Module	Contents	Teaching Hours
No.		(Approx.)
	Basics of Inventory Models (IM): Necessity of IM,	
	Inventory costs, Basic definitions, Classification of IM.	
	Inventory Models with Deterministic Demand:	
	(1) Demand rate uniform, production rate infinite,	
	(2) Demand rate non uniform and production rate	
	infinite, (3) Demand rate uniform and production rate	
	finite, (4) Demand rate uniform, production rate	
	infinite & shortages allowed,	
	(5) Demand rate uniform, production rate finite&	
	shortages allowed.	
	Inventory Models with Probabilistic Instantaneous Demand:	
	(1) Instantaneous demand, setup cost zero, stock level	
	discrete and lead time zero	
	(2) Instantaneous demand, setup cost zero, stock level	
	continuous and lead time zero	45
Ι	Inventory Models with Probabilistic Continuous	15
	Demand:	
	(1) Continuous demand, setup cost zero, stock level	
	discrete and lead time zero	
	(2) Continuous demand, setup cost zero, stock level	
	continuous and lead time zero	
	(3) Continuous demand, setup cost zero, stock level	
	discrete with lead time.	
	Inventory Models with Price Breaks:	
	Purchase inventory models with one and two price	
	breaks	
	Inventory Models with different demand functions:	
	Inventory models with price and time dependent	
	demand, Inventory models with price, time and stock	
	dependent demand, Inventory models with seasonal	
	demand.	

Learning Outcomes:

After studying these topics, the students will be able to

- Learn inventory model and its types
- > Understand the inventory models with uniform and non-uniform demand rate
- Know the inventory models with instantaneous/ continuous demand with discrete/ continuous stock level and lead time zero
- > Minimize the total inventory holding costs and ordering costs



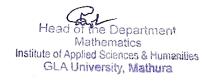
Text Books:

- P. K. Gupta & D. S. Hira, Operations Research, S. Chand Publication, 2014.
- Sven Axsäter, Inventory Control, Springer, 2000.

Reference Book:

E. L. Porteus, Foundations of Stochastic Inventory Theory, Stanford University Press, 2002.





PMAE 0021: RANDOM VARIABLES AND SOFTWARE TOOLS

Course Objectives: To make the students understand the concepts of probability distribution, use of software tools and LaTeX.

Credits: 04

Semester I/II

L-T-P-J: 4-0-0-0

Module No.	Contents	Teaching Hours
		(Approx.)
Ι	Random Variables - Discrete and Continuous, Cumulative Distribution and Probability Density Function, Discrete Random Variables, Probability Mass Function and Cumulative Distribution Function, Expectation of Variables, Moments and Variance, Skewness and Kurtosis, Degenerate and Discrete Uniform Distributions, Bernoulli and Binomial Distribution, Poisson Distribution, Geometric Distribution, Continuous Random Variables and Uniform Distribution, Normal Distribution. Software Tools - Introduction to softwares: Mathematica, Matlab, SPSS and R. Plotting of functions, Symbolic computation, Differentiation and Integration, Numerical solution of equations. Introduction to LaTeX, Syntax, Font Selection, Formatting Text, Page Layout and style, Table preparation, Equation writing, Bibliography with LaTeX.	40

Course Outcomes:

After studying these topics, the students will be able to

CO1: Understand the probability distributions.

CO2: Apply the probability distribution in inventory models.

CO3: Use Mathematica software to get the solution of real-life inventory models.

CO4: Write research article in LaTeX.

Text Books:

- S. C. Gupta and V. K. Kapoor, Fundamentals of Mathematical Statistics: Sultan Chand & Sons, 2014.
- S. Wolfram, The Mathematica book: Wolfram Media, Inc, 2004.
- S. Kottwitz, LaTeX beginner's Guide, Packt Publishing, 2011.

- ➤ G. Grätzer, More Math Into LaTeX, Springer, 2016.
- E. Don, Schaum's Outline of Mathematica and the Wolfram Language, Mc Graw Hill Education, 2018.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	PO1, PO10, , PO11 / PSO1, PSO4
C02	P010, P011 / PS01, PS04
C03	PO3, PO4 / PSO1, PSO2, PSO4
CO4	PO1, PO2, PO10, PO11 / PSO1, PSO3, PSO4



PMAE 0030: PARTIAL DIFFERENTIAL EQUATIONS: METHODS AND APPLICATIONS

Course Objectives: To make the students understand the various analytic and semi analytic approaches to solve linear and nonlinear partial differential equations with given initial and boundary conditions by giving more emphasis to their applications in the real world.

Credits: 04	Cred	its:	04
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Semester I

L-T-P: 4-0-0

Module	Contents	Teaching Hours
No.		(Approx.)
	<u>I Order Linear Partial Differential Equations (PDEs)</u> : Solution by Adomian decomposition method (ADM), The	
	noise terms phenomenon. Solution by the modified	
	decomposition method (MDM) and the variational iteration method (VIM). Method of characteristics,	
	Solution of systems of linear PDEs by ADM and VIM.	
	Heat Flow and Wave Equations by ADM and VIM: Solution of homogeneous and inhomogeneous one	
I	dimensional heat and wave equations by ADM and VIM.	45
I	Solution of higher dimensional heat flow & wave equations by ADM. Solution of Laplace's equation with	
	Dirichlet, Neumann and Robin Boundary conditions by	
	ADM. <u>Non – linear Partial Differential Equations</u> : Calculation	
	of Adomian polynomials, Solution of nonlinear PDEs by	
	ADM, MDM and VIM, Solution of non-linear PDEs systems,	
	Nonlinear advection problem, Goursat problem, Klein– Gordon, Sine–Gordon, Burger's, Telegraph, Schrodinger,	
	Korteweg – deVries (KdV) equations by ADM and VIM.	

Learning Outcomes:

After studying these topics, the student will be able to

- > Understand various numerical methods for solving differential equations
- > Apply the semi analytic methods in real world problems
- Explore new applications of the aforesaid methods
- Solve the non-linear PDE numerically

Text Books:

- > A. M. Wazwaz, Partial Differential Equations and Solitary Wave Theory, Springer, 2009.
- A. M. Wazwaz, Partial Differential Equations: Methods and Applications, Balkema Publishers, 2002.

- G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Academic Publishers, 1994.
- L. Debnath, Non–Linear Partial Differential Equations for Scientists and Engineers, Birkhauser, Springer, 2012.
- > L. C. Evans, Partial Differential Equations, American Mathematical Society, 1998.



Course Curriculum (Session 2021-22) [Ph. D. (Mathematics) Course Work]

PMAE 0031: FRACTIONAL DIFFERENTIAL EQUATIONS

Course Objectives: To make the students understand the concepts of fractional integral, fractional derivative and solution of fractional differential equations.

Credits: 04

Semester I/II

L-T-P-J: 4-0-0-0

Module	Contents	Teaching
No.		Hours
_		(Approx.)
	Special Functions – Euler's functions, Integral functions, One and	
	two parameter Mittag-Leffler functions.	
	<u>Fractional Calculus</u> – Introduction, definition, Fractional integral of	
	order α , Grünwald–Letnikov fractional derivative, Riemann-Liouville	
	(RL) fractional derivative of order α with its properties, Liouville-	
	Caputo fractional derivative of order α with its properties, Laplace	
	transform of fractional integrals and derivatives.	
	Fractional Differential Equations (FDE) – Riemann-Liouville and	
	Caputo fractional differential equations, Existence and uniqueness for	
	the Caputo problem, linear and nonlinear fractional differential	
	equation, Solution by Adomian decomposition method (ADM),	
Ι	Fractional systems of differential equations, Time-fractional and	40
	Space-fractional differential equations.	
	Solution of Fractional Differential Equations – Numerical	
	solution of FDE by fractional variational iteration method (FVIM),	
	Use of FVIM in solving fractional Hunter-Saxton equation, fractional	
	Newell-Whitehead-Segel equation, fractional system of Bloch	
	equations, fractional disease models etc., Hybrid methods for solving	
	fractional differential equations, Application of homotopy perturbation	
	transform method (HPTM) and homotopy analysis transform method	
	(HATM) in solving time-fractional vibration equation, space-fractional	
	Helmholtz equation, time-fractional multi-dimensional telegraph	
	equation etc. Use of any one software package for numerical	
	computation, and drawing figures.	

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Understand the fractional integral and derivatives.
- **CO2**: Apply the numerical methods in solving fractional differential equations.
- **CO3:** Use hybrid methods to get the solution of real-life fractional nonlinear models.

CO4: Identify a research problem.

Text Books:

- C. Milici, G. Draganescu and J. T. Machado, Introduction to Fractional Differential Equations: Nonlinear Systems and Complexity, Springer Nature Switzerland AG, 2019.
- A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, Theory and applications of fractional differential equations, Elsevier B.V., Amsterdam, 2006.

- > I. Podlubny, Fractional differential equations, Vol. 198, Academic Press, USA.
- E. Don, Schaum's Outline of Mathematica and the Wolfram Language, Mc Graw Hill Education, 2018.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	P01, P02, P03, P04 / PS01, PS04
CO2	PO2, PO3, PO4, PO10 / PSO2, PSO4
C03	PO2, PO3, PO4, PO10, PO12 / PSO2, PSO4
CO4	PO4, PO8, PO10 / PSO2, PSO3



Credits: 04

L-T-P: 4-0-0

PMAE 0040: COMPUTER BASED NUMERICAL TECHNIQUES AND SOFT COMPUTING

Course Objectives: To make the students understand the concepts of numerical techniques, artificial neural networks, genetic algorithm and fuzzy logic by giving more emphasis to their applications in soft computing.

Semester I

Module No.	Contents	Teaching Hours (Approx.)
Ι	 Introduction: Brief review & updates of concepts & applications of computer and computer programming, Modern trends. Numerical Techniques: Finding roots of equation & optimization, Least-square-fitting, Laplace-Transform and its applications for the numerical solution of differential equations, FEM and its applications. Neural Networks: Introduction to soft computing & Neural computing, Fundamentals of artificial neural network (ANN), Models of ANN, ANN models: Rosenblatt's perception, McCulloch & Pitts Model, Single layer perceptron, Learning methods in perceptron, Linearly separable task and XOR problem. Multi-Layer Perceptron: Back propagation learning algorithm, Associative memory, Hopfield network, Auto associative memory, Bidirectional hetro-associative memory, ADALINE, MADALINE network, Applications of neural network. Fuzzy Logic: Introduction to Fuzzy and Crisp sets, Fuzzy membership and Fuzzy operations, Properties of fuzzy sets, Linguistic hedges, Fuzzy logic – T-norms and other aggregation operators, Crisp and Fuzzy relations, Fuzzy system, Crisp logic, Propositional logic and its Laws, Inference in propositional logic, Fuzzy logic. Genetic Algorithm (GA): Introduction to GA, Search optimization method, Evolutionary algorithm working principles of GA, Encoding, operators of GA, Crossover and mutation, Basics of genetic algorithm with example for maximize, Introduction to genetic programming and Basic concepts. 	45

Learning Outcomes:

After studying these topics, the student will be able to

- > Implement code for numerical methods to find the solution of non-linear equations or the system of linear equations
- > Develop code for numerical methods to estimate the solution of higher order ordinary differential equations or systems of first order ordinary differential equations
- Construct graphical displays of science/engineering data and interpret the role of such displays in data analysis



- > Analyze various neural network architectures
- Define the fuzzy systems
- > Analyze the genetic algorithms and their applications

Text Books:

- S. Rajsekaran and G. A. Vijayalakshmi Pai, Neural Networks, Fuzzy Logic and Genetic Algorithm: Synthesis and Applications, Prentice Hall of India, 2011.
- Y. P. Kanetkar, Let us C, BPB publications, 2004.
- > V. Rajaraman, Computer oriented numerical methods, PHI Learning Pvt. Ltd., 2018.

- > T. J. Ross, Fuzzy Logic with Engineering Applications, John Wiley and Sons, 2016.
- D. E. Goldberg, Genetic Algorithm in Search Optimization and Machine Learning, Adission-Wesley, 2016.
- Karray, Soft Computing and Intelligent Systems Design: Theory, Tools and Applications, Pearson Education, 2009.



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PMAE 0041: APPLIED STATISTICS AND OPTIMIZATION TECHNIQUES

Course Objectives: To make the students understand the concepts of statistics and optimization techniques by giving more emphasis to their applications in soft computing.

Credits: 04

Semester I

L-T-P: 4-0-0

Module	Contents	Teaching Hours
No.		(Approx.)
Ι	 Random Variables: Definition, Cumulative distribution function, Continuous, Discrete and Mixed random variables, Probability density function, Probability mass function. Moments of random variables, Mean and variance of random variable, Coefficients of variation, Skewness and Kurtosis, Moments, Covariance and Correlation coefficient. Properties of distribution functions. Probability Distributions and Testing of Hypothesis: Binomial, Poisson and Normal distributions. Introduction to Statistical analysis, Population distribution, Sampling and Non sampling Errors, Point estimation, Interval estimation, Student's t-distribution, Chi Square distribution, F-distribution. Optimization Techniques: Introduction, Classification of Optimization problems, Classical optimization techniques: Single variable optimization, Multivariable optimization with no constraints, Multivariable optimization with equality constraints. Non-linear Programming: Minimization methods, Introduction, Exhaustive search, Direct root method, Newton method, Secant method, Constrained optimization techniques, Gradient method, Constrained optimization techniques, Processes; Definition and Classification of Stochastic processes, Poisson process, Birth and Death process, Applications to queues, Discrete time Markov chains. 	<u>(Approx.)</u> 45

Learning Outcomes:

After studying these topics, the students will be able to

- Apply the concepts of probability, random variables and operation research in the area of computer networks, image processing etc.
- > Use statistical methods for the design and analysis of computer simulation experiments
- State and define the inference from small samples including difference between two means, paired differences, population variances, and two population variances
- Understand the characteristics of different types of decision-making environments and the appropriate decision-making approaches
- > Learn Poisson process and its applications in queuing theory



Text Books:

- K. S. Trivedi, Probability and Statistics with Reliability, Queuing and Computer Science Applications, Wiley, 2016.
- H. A. Taha, Operations Research An Introduction, Prentice Hall, 2011.

- Papoulis and S. U. Pillai, Probability, Random Variables and Stochastic Processes, Tata McGraw Hill, 2017.
- R. M. Gray and L. D. Davisson, An Introduction to Statistical Signal Processing, Cambridge University Press, 2004.
- S. L. Miller and D. G. Childers, Probability and Random Processes, Academic Press, 2012.
- Y. Viniotis, Probability and Random Processes for Electrical Engineers, McGraw Hill, 1998.
- S. D. Sharma, Computer Based Optimization Techniques , Kedr Nath Publication, 2011.
- K. Swaroop and Man Mohan, Operation Research, Sultan Chand & Sons, 2011.
- > D. S. Hira and P. K. Gupta, Operation Research, S. Chand Publication, 2005.



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PMAE 0050: DIFFERENTIAL GEOMETRY OF MANIFOLDS

Course Objectives: To make the students understand the concepts of surface, tangent vectors, tangent plane, curvature and differential manifold by giving more emphasis to their applications in differential geometry.

Credits: 04

Semester I

L-T-P: 4-0-0

Module	Contents	Teaching Hours
No.		(Approx.)
	Functions on Euclidean spaces, Continuity, Differentiability,	
	Partial and Directional derivatives, Chain rule, Inverse	
	function theorem, Implicit function theorem, Smooth	
	Urysohn lemma, Partition of unity, Change of variables.	
	Regular surfaces in R ³ , Coordinate neighbourhoods, Tangent	
	vectors, Tangent plane, Normal fields, Orientability,	
	Examples of surfaces, Level sets of smooth functions on R^3 .	
Ι	Smooth functions on surfaces, Differential of a smooth	45
	function, Gauss map, Shape operator (or Weingarten map),	
	Normal sections, Principal curvatures, Gaussian and Mean	
	curvature, Theorema Egregium, Isometries of surfaces.	
	Differential manifolds, Differential functions on manifolds,	
	Tangent spaces, Vector fields, Differential forms on	
	manifolds, Orientations, Integration on manifolds, Stoke's	
	theorem on manifolds	

Learning Outcomes:

After studying these topics, the student will be able to

- Understand change of variables
- > Know regular surfaces and orientability
- > Use Weingarten map, principal curvatures and isometries of surfaces
- > Solve the problems based on Gauss map, normal sections, Gaussian and mean curvature
- > Learn the concept of differential forms on manifolds and integration on manifolds

- > M. Spivak, Calculus on Manifolds, Addison-Wesley, 1965.
- J. R. Munkers, Analysis on Manifolds, Addison-Wesley, 1991.
- > A. Pressley, Elementary Differential Geometry, Springer, 2001.
- A. Gray, Modern Differential Geometry of Curves and Surfaces with Mathematica, CRC Press, 1997.



Head of the Department Mathematics Institute of Applied Sciences & Humanities GLA University, Mathura

PMAE 0051: RIEMANN-FINSLER GEOMETRY

Course Objectives: To make the students understand the concepts of Finsler metric, Chern connection, flag curvature, Riemann curvature, Parallel translation and Jacobi field by giving more emphasis to their applications in Finsler geometry.

Credits: 04

Semester II

L-T-P: 4-0-0

Module	Contents	Teaching Hours
No.		(Approx.)
	Minkowski norm, Finsler metric, Length Structure and Volume Form, Randers metric. Chern connection, Flag Curvature, Finsler metric of constant flag curvature, Finsler metric of scalar flag curvature. Projective Finsler metric, Projective flat Finsler metrics, Parallel vector Fields, Parallel translation, Landsberg metric, Berwald metric. Riemann curvature, S-Curvature, Isotropic S-curvature, Exponential map, First and second variation, Jacobi field.	-
	Maple programs for Spray Coefficients of two dimensional Finsler metric, Gauss curvature, Spray Coefficients of (α, β) metric.	

Learning Outcomes:

After studying these topics, the student will be able to

- > Know Finsler metric, Randers metric and Chern connection
- > Understand Finsler metric of constant and scalar flag curvatures.
- > Solve the problems based on projective and projective flat Finsler metrics
- Learn Landsberg and Berwald metrics
- Find parallel vector fields, parallel translation, Riemann curvature, S-curvature and Exponential map
- > Calculate spray coefficients and Gauss curvature using Maple

- Z. Shen, Differential Geometry of Spray and Finsler Space, Kluwer Academic publishers, 2001.
- Z. Shen, Lectures on Finsler Geometry, Lectures on Finsler Geometry, World Scientific, 2001.
- S. S. Chern and Z. Shen, Riemannian Finsler Geometry, World Scientific, 2004.
- D. Bao, S. S. Chern and Z. Shen, An Introduction to Riemannian-Finsler Geometry, GTM, Springer, 2000.
- P. L. Antonelli, R. S. Ingarden, M. Matsumoto, The Theory of Sprays and Finsler Spaces with Application in Physics and Biology, Kluwer Academic Publishers, 1993.
- M. Matsumoto, Foundations of Finsler Geometry and Special Finsler spaces, Kaiseisha Press, Japan 1986.



Course Curriculum (Session 2020-21) [Ph. D. (Mathematics) Course Work)]

PMAE 0060: FLUID DYNAMICS AND SOFTWARE TOOLS

Course Objectives: To make the students understand the concepts of fluid, its properties and behavior under various conditions of internal and external flows.

Module	Contents	
No.	contents	Teaching Hours (Approx.)
Ι	 Basic Concepts of Fluid Dynamics: Real fluids and ideal fluids, stream lines, steak lines and path lines, steady and unsteady flows, Lagrangian and Eulerian description, fundamental principles of conservation, conservation of mass, equation of continuity, Reynolds transport theorem. Governing Equations of Fluid Dynamics: The continuity equation, momentum equation, energy equation, conservation of linear momentum, Navier-Stokes equation, conservation of energy, general scalar transport equation, Boussinesq approximation, some solvable fully developed viscous flow: (i) steady flows between parallel plates, (ii) steady flow in a pipe, (iii) steady flows between concentric cylinders. Dimensional Analysis Reynolds law of similarity, physical signification of some non-dimensional parameters: Prandtl number, Mach number, Reynolds number, Rayleigh number, Grashof number, Nusselt number, Laminar steady flow of incompressible viscous fluids in tubes of circular and rectangle cross- section. Turbulence and its Modelling: Characteristics of turbulence, effect of turbulent fluctuations on mean flow, turbulent flow calculations, turbulence modelling. Software Tools: Introduction to software: MATHEMATICA and MAPLE. Introduction to LaTeX, Syntax, Font Selection, Formatting Text, Page Layout and style, Table preparation, Equation writing, Bibliography with LaTeX. 	<u>40</u>

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Understand physical properties of a fluid.
- **CO2**: Know the classification of the basic equations of fluid dynamics.
- **CO3:** Recognize the type of fluid flow occurring in a particular physical system and to use the appropriate model equations to investigate the flow.
- **CO4:** Learn the dimensional analysis.
- **CO5:** Write the research article in LaTeX.

Text Books:

- > P. G. Drazin and W. H. Reid, Hydrodynamic Stability, Cambridge University Press, 2004.
- S.V. Parankar, Numerical Heat Transfer and Fluid Flow, McGraw-Hill. 1980.
- K. Muralidhar and G. Biswas, Advanced Engineering Fluid Mechanics, Narosa Publishing House, 2006.
- G. D. Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods, Clarendon Press, 1985.
- S. Wolfram, The Mathematica book: Wolfram Media, Inc, 2004.
- S. Kottwitz, LaTeX beginner's Guide, Packt Publishing, 2011.

Reference Books:

> H. Schlichting, Boundary Layer Theory, McGraw Hill Education, 2014.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	P01, P02, P04, P06, P010, / PS01, PS02, PS04
CO2	P01, P02, P03, P04, P06, P09, P012 / PS01, PS02, PS04
C03	P01, P02, P03, P04, P05, P06, P010, P012 / PS01, PS02, PS03, PS04
CO4	P01, P02, P03, P04, P010, P011, P012/ PS01, PS02, PS04
C05	P01, P02, P010, P011 / PS01, PS03, PS04



Course Curriculum (Session 2020-21) [Ph. D. (Mathematics) Course Work)]

PMAE 0061: COMPUTATIONAL METHODS FOR FLUID DYNAMICS

Course Objectives: To make the students understand the concepts of stepwise procedure to completely solve a fluid dynamics problem using computational methods.

Credits: 04

Semester I/II

L-T-P-J: 4-0-0-0

Module No.	Contents	Teaching Hours
NO.		(Approx.)
Ι	Introduction to Computational Fluid Dynamics (CFD): Advantages and applications of computational field, governing equations for CFD, continuity equation, momentum equation, energy equation, additional equations for turbulent flow. CFD Techniques: Discretization of governing equations, Finite-difference method, finite-volume method, converting governing equation to algebraic equation system, numerical solutions to algebraic equations-Direct method, iterative methods, pressure velocity coupling. Hydrodynamic Stability: Mathematical formulation of the stability problem of incompressible flow, method of normal modes, Squire's theorem and Orr-Sommerfeld equation; concept of boundary layers, fluid flow through a porous medium; porosity, permeability, Darcy law and its extensions.	40

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Understand CFD technique.
- **CO2**: Demonstrate the ability to analyze a flow field to determine various quantities of interest, such as flow rates, heat fluxes, pressure drops, losses, etc., using flow visualization and analysis tools.
- CO3: Learn the hydrodynamics stability
- **CO4:** Simulate simple CFD models and analyze its results.

Text Books:

- J. Tu, G. Heng Yeoh and C. Liu Computational Fluid Dynamics: A practical Approach, Butterworth-Heinemann, 2018.
- S.V. Parankar, Numerical Heat Transfer and Fluid Flow, McGraw-Hill. 1980.
- > T. J. Chung, Computational Fluid Dynamics, Cambridge University Press, 2010.
- J. H. Ferziger and M. Peric, Computational Methods for Fluid Dynamics, Springer, 2002.
- G. D. Smith, Numerical Solution of Partial Differential Equations: Finite Difference Methods, Clarendon Press, 1985.

- H. K. Versteeg and W. Malalasekera, An Introduction to Computational Fluid Dynamics: The Finite Volume Method, Pearson, 2010.
- J. C. Tannehill, D. A. Anderson and R. H. Pletcher, Computational Fluid Mechanics and Heat Transfer, McGraw Hill, 2002.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	PO1, PO2, PO4, PO5, PO6, PO10, PO11/ PSO1, PSO2, PSO4
C02	PO1, PO2, PO3, PO4, PO6, PO9, PO12 / PSO1, PSO2, PSO4
CO3	PO1, PO2, PO3, PO4, PO5, PO6, PO12 / PSO1, PSO2, PSO4
C04	PO1, PO2, PO3, PO4, PO10, PO11, PO12/ PSO1, PSO2, PSO4



Course Curriculum (Session 2020-21) [Ph. D. (Mathematics) Course Work)]

PMAE 0070: FOUNDATIONS OF COMPLEX ANALYSIS-I

Course Objectives: To make the students understand the foundations for functions of a complex variable and their applications.

Credits: 04 Semester I L-T-P-J: 4-0-0-0

Module No.	Contents	Teaching Hours (Approx.)
I	Functions of a complex variable, Spherical representation of complex numbers, Limits, Theorems on limit, Continuous functions, Differentiability, Cauchy-Riemann equations, Polar form of Cauchy Riemann equations, Analytic functions, Harmonic functions. Definite integral, Line integral, Cauchy's theorem, Morera's theorem, Cauchy's integral formula, Higher derivatives, Liouville's theorem, Fundamental theorem of algebra, Maximum modulus theorem. Sequences and Series, Sequences and Series of functions, Power series, Taylor's theorem, Singularities, Laurent's theorem, Residues, Exponential and trigonometric functions, Contour integrations. Computer Skills: Introduction to LaTeX and MAPLE, Power point presentation through beamer.	40

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Use the information needed to prove theorems and establish mathematical results.
- **CO2**: Demonstrate the ability to integrate knowledge and ideas of complex differentiation and complex integration in a coherent and meaningful manner and use appropriate techniques.
- **CO3:** Demonstrate the ability to think critically by proving mathematical conjectures and establishing theorems from complex analysis.
- **CO4:** Use Latex in article writing and power point presentation.

Text Books:

- V. Ruel Churchill and J. W. Brown, Complex Variables and Applications, McGraw-Hill Publishing Company, 2013.
- S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 2011.
- > H. A. Priestly, Introduction to Complex Analysis, Clarendon Press, 2006
- J. B. Conway, Functions of one Complex Variable, Springer-Verlag, Narosa Publishing House, 1995.
- L. V. Ahlfors, Complex Analysis, McGraw Hill Education, 2017.

- S. Lang, Complex Analysis, Addison Wesley, 1977.
- M. J. Ablowitz and A. S. Fokas, Complex Variables: Introduction and Applications, Cambridge University Press, 2003.
- > W. Rudin, Real and Complex Analysis, Mc Graw Hill Education, 2017.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs
C01	PO1, PO2, PO4, PO5, PO6, PO10, PO11/ PSO1, PSO2, PSO4
CO2	PO1, PO2, PO3, PO4, PO6, PO9, PO12 / PSO1, PSO2, PSO4
CO3	PO1, PO2, PO3, PO4, PO5, PO6, PO12 / PSO1, PSO2, PSO4
CO4	PO1, PO2, PO3, PO4, PO10, PO11, PO12/ PSO1, PSO2, PSO4



Course Curriculum (Session 2021-22) [Ph. D. (Mathematics) Course Work)]

PMAE 0071: FOUNDATIONS OF COMPLEX ANALYSIS-II

Course Objectives: To make the students understand the concepts of contemporary complex analysis and to use the methods of complex analysis in other areas of mathematics.

Credits: 04

Semester I/II

L-T-P-J: 4-0-0-0

Module No.	Contents	Teaching Hours (Approx.)
Ι	Linear Functions, Function 1/z, Bilinear transformations, their properties and classification, Functions z ² and z ^{1/2} , Transformations w=exp(z) and w= sin z, Conformal mappings, Harmonic conjugate, Transformation of Harmonic Functions, Transformation of Boundary Conditions, Steady temperatures, Steady temperature in a half plane and related problems. Schwarz's lemma, Meromorphic functions, Argument principle, Rouche's theorem, Inverse function theorem. Open mapping theorem and Hurwitz' theorem, Riemann mapping theorem (Statement only), Area Theorem, Biberbach Theorem and conjecture, Koebe ¹ / ₄ theorem. Univalent functions, Star-like, Convex and Close-to-Convex functions.	40

Course Outcomes:

After studying these topics, the students will be able to

- **CO1:** Use the information needed to prove theorems and establish mathematical results.
- **CO2**: Learn the basic techniques of contemporary complex analysis as well as use of these techniques in various applications such as harmonic analysis, univalent functions theory and special functions.
- **CO3:** Demonstrate the ability to think critically by proving mathematical conjectures and establishing theorems from complex analysis.
- **CO4:** Demonstrate the ability to integrate knowledge and ideas of Meromorphic functions Univalent functions and Riemann mapping theorem in a coherent and meaningful manner and use appropriate techniques.

Text Books:

- V. Ruel Churchill and J. W. Brown, Complex Variables and Applications, McGraw-Hill Publishing Company, 2013.
- S. Ponnusamy, Foundations of Complex Analysis, Narosa Publishing House, 2011.
- > H. A. Priestly, Introduction to Complex Analysis, Clarendon Press, 2006
- > P. L. Duren, Univalent Functions, Springer-Verlag, New York, 1983.
- A. W. Goodman, Univalent Functions, Mariner Publishing Company, Michigan, 1983
- L. V. Ahlfors, Complex Analysis, McGraw Hill Education, 2017.

- S. Lang, Complex Analysis, Addison Wesley, 1977.
- M. J. Ablowitz and A. S. Fokas, Complex Variables: Introduction and Applications, Cambridge University Press, 2003.
- > W. Rudin, Real and Complex Analysis, Mc Graw Hill Education, 2017.



Mapping of Course Outcomes (COs) with Program Outcomes (POs) and Program Specific Outcomes (PSOs):

COs	POs/ PSOs	
C01	PO1, PO2, PO4, PO6, PO10, PO11/ PSO1, PSO2, PSO4	
CO2	P01, P02, P03, P04, P06, P09, P010, P011, P012 / PS01, PS02, PS04	
CO3	P01, P02, P03, P04, P05, P06, P010, P011, P012 / PS01, PS02, PS03, PS04	
CO4	P01, P02, P03, P04, P010, P011, P012/ PS01, PS02, PS04	